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Published in:
Physics Letters B

DOI:
[10.1016/0370-2693\(77\)90860-7](https://doi.org/10.1016/0370-2693(77)90860-7)

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
1977

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Arima, A., Ohtsuka, T., Iachello, F., & Talmi, I. (1977). Collective nuclear states as symmetric couplings of proton and neutron excitations. *Physics Letters B*, 66(3), 205-208. [https://doi.org/10.1016/0370-2693\(77\)90860-7](https://doi.org/10.1016/0370-2693(77)90860-7)

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COLLECTIVE NUCLEAR STATES AS SYMMETRIC COUPLINGS OF PROTON AND NEUTRON EXCITATIONS

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Received 30 November 1976

Collective nuclear states are described by symmetric couplings of proton and neutron pairs. These $J=0$ and $J=2$ pairs are represented by s - and d -bosons respectively. The multiplet structure of the combined system is given by representations of the $SU(6) \times SU(2)$ group, the Arima-Iachello interacting bosons corresponding to the fully symmetric ones. The validity of the boson picture is attributed to the attractive proton-neutron interaction which is also responsible for the transition from vibrational to rotational spectra.

Recently a promising approach to collective states of nuclei has been proposed [1, 2] which is quite simple and yet can describe the situation in many nuclei. It can describe very well the vibrational and rotational limits of collective spectra as well as the transition between them. Collective states of even even nuclei are constructed as states of N bosons which can occupy a level with $L=0$ (s -bosons) or another with $L=2$ (d -bosons). The Hamiltonian includes one boson terms and two-boson interactions (hence the name — Interacting Boson Approximation — IBA). Single boson operators are written in terms of boson creation and annihilation operators as follows

$$(s^+s)^{(0)}, (d^+ \times d)^{(0)}, (d^+ \times d)^{(1)}_{\mu}, (d^+ \times d)^{(2)}_{\mu}, \quad (1) \\ (d^+ \times d)^{(3)}_{\mu}, (d^+s)^{(2)}_{\mu} \text{ and } (s^+d)^{(2)}_{\mu}.$$

The single boson terms in the Hamiltonian are the scalars in (1) and two-boson interactions are scalar products of operators in (1), all multiplied by appropriate coefficients. Thus, the total number of bosons ($N = n_s + n_d$) is conserved whereas some of the interactions do not conserve separately n_d (and n_s). The 36 operators (1) are the generators of $U(6)$ — the group of unitary transformations in the space spanned by the one-boson

states $s^+|0\rangle, d^+_{\mu}|0\rangle$. As a result, all eigenstates are *symmetric* irreducible representations $[N]$ of $SU(6)$ in this space.

In the vibrational limit, interactions which change s -bosons into d -bosons and vice versa are ignored in comparison with other terms and in particular with the single boson energy difference $\epsilon = \epsilon_d - \epsilon_s$. The Hamiltonian is then constructed from those operators in (1) that conserve n_d (and n_s). These are the $25 + 1$ generators of $U(5) \times U(1)$ and the eigenstates, which correspond to anharmonic vibrator, have been discussed in details in refs. [1, 2]. As ϵ goes to zero the rotational limit may be reached. In that limit H is constructed from the operators

$$(s^+s)^{(0)} + \sqrt{5}(d^+ \times d)^{(0)}, (d^+ \times d)^{(1)}_{\mu} \quad (2) \\ \text{and}$$

$$(d^+s + s^+d)^{(2)}_{\mu} - (\sqrt{7}/2)(d^+ \times d)^{(2)}_{\mu},$$

which are the generators of $U(3)$. Hence, the eigenstates are characterized by the irreducible representations of $SU(3)$ in the space of s - and d -bosons. Thus, $SU(3)$ symmetry which was shown to give a good description of rotational spectra is obtained in IBA independently of any degeneracies of single nucleon energies.

The success of IBA poses many interesting questions, some of which concern its relationship with the collective model of strongly deformed nuclei [3]. Here we deal with the relations between IBA and the shell model [4, 5]. We try to understand the structure of states with s- and d-bosons in terms of nucleon states and more specifically in terms of *proton and neutron states*. We will show that the strong interaction between protons and neutrons is the physical basis of IBA. We will also show that in the boson picture a simple interaction can be constructed which gives rise to the decrease of ϵ and the transition from vibrational to rotational spectra.

The conservation of the total number of bosons $N = n_s + n_d$ strongly suggests that they should represent (particle-particle) nucleon pairs. For *identical* valence nucleons (for instance, protons) there exists a simple realization of s-bosons. Consider the pair creation operators [6]

$$S_p^+ = \sum_j \alpha_j S_j^+, \quad S_j^+ = \frac{1}{2} \sum_m (-1)^{j-m} a_{jm}^+ a_{j-m}^+, \quad (3)$$

where the range of j is in major shell. If the fermion Hamiltonian satisfies

$$H_p S_p^+ |0\rangle = V_p S_p^+ |0\rangle \text{ and } [[H_p, S_p^+], S_p^+] = \Delta_p (S_p^+)^2, \quad (4)$$

we obtain for any number m

$$H_p (S_p^+)^m |0\rangle = (m V_p + \frac{1}{2} m(m-1) \Delta_p) (S_p^+)^m |0\rangle. \quad (5)$$

The states (5) describe very well ground states of semi-magic nuclei. The fermion pair creation operators S_p^+ satisfy with H_p the same commutation relations as those satisfied by the boson creation operators s^+ with that part of the IBA Hamiltonian which contains only s-boson operators, i.e.

$$(V_p - \frac{1}{2} \Delta_p) (s^+ s)^{(0)} + \frac{1}{2} \Delta_p (s^+ s)^{(0)} (s^+ s)^{(0)}. \quad (6)$$

It should be stressed that there are shell model Hamiltonians with single nucleon and two nucleon operators which satisfy the relations (4) [6]. We do not propose to express such shell model Hamiltonians in terms of complicated expansions of boson operators. Also no attempt is made to express S_p^+ in terms of "boson expansions". The correspondence between operators S_p^+ and s^+ is only in their commutation relations with their Hamiltonians. The Hamiltonian (6) with boson creation and annihilation operators, should be considered as a simple model replacing for ground states, the more

complicated and detailed shell model problem;

Shell model realizations of d-bosons are more complicated. In ref. [6] creation operators of nucleon pairs in $J=2$ states were also considered

$$D_\mu^+ = \sum_{jj'} \beta_{jj'} D_{jj',\mu}^+, \quad D_{jj',\mu}^+ = \sum_{mm'} (jmj' m' | jj' 2\mu) a_{jm}^+ a_{j'm'}^+. \quad (7)$$

Conditions under which $(S_p^+)^m D_\mu^+ |0\rangle$ are eigenstates were stated but the D_μ^+ operators (7) do not simply correspond to boson creation operators d_μ^+ . For example, the state $(D^+ \times D^+)^{(0)} |0\rangle$ is, in general, not orthogonal to the state $(S_p^+) |0\rangle$ which corresponds to the state of two s-bosons.

We can still consider shell model *states* which correspond to the states $(s^+)^{n_s} (d^+)^{n_d}_{\gamma JM} |0\rangle$ with n_s s-bosons and n_d d-bosons ($n_s + n_d = N$) coupled to a total J and M (γ stands for the additional necessary quantum numbers [1, 2]). Consider all states $(S_p^+)^{n_s} (D^+)^{n_d}_{\gamma JM} |0\rangle$ obtained by replacing boson operators s^+ and d_μ^+ by fermion pair operators S_p^+ and D_μ^+ respectively. Let us now subtract from each of them all components of the form $(S_p^+)^{n_s+1} B_{JM}^+ |0\rangle$ where B_{JM}^+ are all operators creating $2(n_d - 1)$ nucleons. The non-vanishing states obtained this way are characterized by n_d, γ, JM . These states turn out to be independent but they need not be orthogonal. Still in actual cases [4] an orthogonalized set can be found whose states have large overlaps with states of the original set. Hence, the states can be still labelled by the quantum numbers γ . The states with different values of n_d are orthogonal to each other and thus correspond to boson states.

For a single j -shell, or for equal values of α_j in (3), the states thus obtained have definite (nucleon) seniorities given by $\nu = 2n_d$. In fact, in the case of a single j -shell, it is possible [4] to construct explicitly the nucleon states which correspond to boson states. The operators used in that construction create two nucleons coupled to $J=2$ but their structure in terms of fermion creation and annihilation operators is more complex than (7).

Here we shall not be further concerned with the shell model (or microscopic) description of s- and d-bosons. Rather, we shall now use the boson picture to consider nuclei with both valence protons and valence neutrons outside closed shells. We will show how states of the combined system can be constructed in the model of s- and d-bosons of valence protons and valence neutrons. The Hamiltonian H will be the sum

$$H = H_p + H_n + V_{pn}, \quad (8)$$

where H_p and H_n are identical with the IBA Hamiltonian in which the operators create and annihilate *proton* bosons and *neutron* bosons respectively. The proton-neutron interaction V_{pn} is a two boson operator which is a linear combination of scalar products of operators (1) for protons and operators (1) for neutrons. This guarantees that the numbers of proton bosons and the number of neutron bosons are separately conserved.

In order to find the state which corresponds to $(s^+)^N|0\rangle$ of IBA we consider the vibrational limit where it is the ground state. If there are $2m$ valence protons and $2n$ valence neutrons then $N = m + n$ (if more than half of the proton major shell is filled we take $2m$ to be the number of *proton holes* and similarly for the neutrons). In that case (in the vibrational limit) the normalized ground state will be given by

$$|\hat{0}\rangle = \frac{(s_p^+)^m (s_n^+)^n}{\sqrt{m!} \sqrt{n!}} |0\rangle. \quad (9)$$

The eigenvalue of the relevant part of H (with only s-boson operators) in the state (9) is

$$E_0 = \mu_p m + \nu_p \frac{1}{2} m(m-1) + \mu_n n + \nu_n \frac{1}{2} n(n-1) + \nu_{pn} mn, \quad (10)$$

(where $\mu_p, \nu_p, \mu_n, \nu_n$ replace $V_p, \Delta_p, V_n, \Delta_n$).

In the absence of V_{pn} the lowest states with $J=2$ would be $d_p^+ (s_p^+)^{m-1} (s_n^+)^n |0\rangle$ and $(s_p^+)^m d_n^+ (s_n^+)^{n-1} |0\rangle$. The proton-neutron interaction strongly admixes these two states so that one linear combination becomes lower. A particularly simple linear combination is

$$\begin{aligned} |\hat{1}\rangle &= \frac{1}{\sqrt{m+n}} (d_p^+ s_p + d_n^+ s_n) |\hat{0}\rangle \\ &= \frac{1}{\sqrt{(m+n)m!n!}} (d_p^+ s_p + d_n^+ s_n) (s_p^+)^m (s_n^+)^n |0\rangle. \end{aligned} \quad (11)$$

If V_{pn} has the desired properties which will be discussed below, the state (11) will be an eigenstate with considerably lower energy than the $J=2$ excitations of valence protons or valence neutrons (in semimagic nuclei).

A strong proton-neutron interaction will probably not have states of the vibrational limit as eigenstates. However, in the boson model, it is possible to construct a special quadrupole interaction between bosons defined by

$$B = T^+ \cdot T = (d_p^+ s_p + d_n^+ s_n) \cdot (s_p^+ d_p + s_n^+ d_n), \quad (12)$$

which conserves the total number of proton and neutron d-bosons. A straightforward calculation shows that both (11) and (9) are eigenstates of B as well as all states of the form

$$(d_p^+ s_p + d_n^+ s_n)^n \gamma_{JM} |\hat{0}\rangle, \quad (13)$$

where the n_d operators T^+ are coupled to angular momentum J and γ stands for the additional necessary quantum numbers [1, 2]. We identify the states (13) as IBA states with n_d d-bosons (and $n_s = N - n_d$ s-bosons). In particular the states (9) correspond to IBA states with s-bosons only ($n_s = N$). The eigenvalues of B in the states (13) are

$$(m+n)n_d + (-2)\frac{1}{2}n_d(n_d-1) = Nn_d - n_d(n_d-1). \quad (14)$$

In (14), the quadratic term is a constant interaction between pairs of d-bosons whereas the linear term contributes to the single d-boson energy. If H includes the term $-\beta B$, then the single d-boson energy is

$$\epsilon = \epsilon_0 - \beta N, \quad (15)$$

and, with appropriate choice of β , decreases linearly with the number of valence protons and neutrons. Detailed calculations show that if such a linear decrease of ϵ is introduced into the IBA Hamiltonian, good agreement can be obtained with the observed low lying levels in Sm isotopes [7]. The strong and attractive proton-neutron interaction is thus responsible for the existence of the eigenstates (11) and (13). We believe that also in actual nuclei it is responsible for the lowering of ϵ and the transition to rotational spectra.

We now consider the general properties of H which make (13) and (9) eigenstates. These eigenstates have a certain symmetry between proton and neutron excitations which should follow from certain symmetry properties of H . The symmetry of the Hamiltonian and corresponding symmetry properties of its eigenstates will now be discussed.

We assign proton bosons and neutron bosons $+\frac{1}{2}$ and $-\frac{1}{2}$ projections of a formal spin vector to be called F -spin. It is analogous, but definitely *not identical* with isospin (in nuclei with valence protons and neutrons in different major shells *all* our states have definite isospin $T = |Z - N|/2$). Using the F -spin formalism we can build all states as fully symmetric in the proton *and* neutron bosons (and not just in each of them separately).

If in the Hamiltonian the interactions in any given allowed state between proton-proton, proton-neutron and neutron-neutron bosons are equal, H commutes with the F -spin operators (it is a scalar with respect to F -spin). The eigenstates of H can then be characterized by their symmetry in the boson space and therefore by their symmetry in F -spin space. The complete wave function Ψ can be written as a sum of products of functions ϕ_k in the s- and d-boson space (these form bases of irreducible representations of $SU(6)$) and functions Ω_k in the F -spin space (forming bases for $SU(2)$ representations). The Young tableaux characterising the symmetry of the F -spin functions have at most two rows. The requirement that the total wave functions $\Psi = \sum \phi_k \Omega_k$, which form representations of $SU(6) \times SU(2)$, must be fully symmetric imposes on the functions ϕ_k and Ω_k the *same* type of symmetry. Therefore, the $SU(6)$ representations have also at most two rows with lengths n_1 and n_2 which are determined by N and total F -spin value

$$n_1 = \frac{1}{2}N + F, \quad n_2 = \frac{1}{2}N - F. \quad (16)$$

Thus, the total F -spin value is a good quantum number. States with fully symmetric functions in the boson space belong to diagrams with $n_1 = N$, $n_2 = 0$ and thus, have $F = N/2$. These states are expected to have in actual cases the lowest energy. In addition, there are states with functions ϕ_k which have lower (or mixed) symmetry. The latter are characterized by lower F -spin values, $0 \leq F \leq N/2$. The higher the value of F , the higher the symmetry and in actual cases, the lower the energy.

Coming back to the states discussed above and considering them from the F -spin point of view, we notice that (9) must be fully symmetric since all bosons are in the same $L=0$ level. Furthermore, the operator T^+ is a scalar with respect to F -spin and thus all states (13) are also fully symmetric in the boson space and have $F = N/2$. *The IBA scheme of states (that of $SU(6)$) is an appropriate scheme for the low-lying collective states if the Hamiltonian of the combined system (of valence*

protons and neutrons) does not admix states with different F -values. We emphasize that H need not be a scalar with respect to F -spin. It could well include a term like (10) which is a function of N and M_F . This term (10), which is important for ground state energies, takes care of most differences in interaction energies between protons and neutrons which are in different major shells. Even the Coulomb interaction may be reasonably well approximated by the proton part in (10). If E_0 (given by (10)) is removed from the Hamiltonian the differences between protons and neutrons are only in their excitations and those are not very different. The rest of H could then be approximated by a scalar in F -spin.

In conclusion, it is worthwhile mentioning that by considering proton and neutron states corresponding to s- and d-boson states we have selected a restricted set of states out of the full (and usually very large) shell model space. The IBA states from a sub-set characterized by the fully symmetric representations $[N]$. These states appear to be lowest in energy and to have the collective properties observed in nuclei.

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